CALCULATIONS FOR the CELESTIAL OBSERVATIONS that MERIWETHER LEWIS made at the THREE FORKS OF THE MISSOURI (Point of Observation No. 39 for 1805)

July 28 - 29, 1805

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FOREWORD

While Meriwether Lewis was at Camp Island near the Three Forks of the Missouri he spent many hours preparing for and taking celestial observations. From all those observations, however, the only calculations he made were for latitude from the sun's noon altitude, July 28 and 29. Lewis's calculated latitudes, unfortunately, are nearly 30 miles farther south than the true latitude of Camp Island Camp. This difference in latitude results from his using the wrong index error for the octant. The other observations were not calculated until nearly two hundred years after Lewis took them; those observations were:

1) two sets of a.m and p.m. Equal Altitudes of the sun to check the chronometer's time,
2) three sets of Lunar Distance observations for longitude (two with the sun, one with Antares) and,
3) three sets of observations (two with the sun, one with Polaris) to determine the variation of the compass needle or magnetic declination.

Before the advent of electronic calculators and computers all the mathematical operations would have been done "long hand." Multiplication, division, powers and roots would have been done by logarithms. All operations using trigonometric functions also would have been performed using logarithms. In addition, the procedures for making the calculations usually were set out in work-sheet forms in books that had been published by mathematicians trained in making the calculations. Thus, the person doing the calculations merely filled in the observational data in the proper places and followed the outlined procedure, step by step – often without understanding the why or wherefore of the mathematical operations.

All the calculations that follow were made using an electronic calculator in a non-program mode, and the operations are set out in step-by-step fashion to help the reader follow the complex operations.

The mathematician of the Lewis-and-Clark era would have used the Nautical Almanac for the year of observation. I also used the Nautical Almanacs for 1803 - 1806 even though there are several excellent computer programs that can recalculate – with greater accuracy and precision than often exists in the almanacs of the time – the needed celestial information. The mathematician of the Lewis-and-Clark era also would have used standard tables such as Tables Requisite to determine the corrections for refraction and parallax and would have used numerous other tables in them to facilitate the tedious, long-hand operations. I did not have access to Tables Requisite for the years of the expedition, but used modern formulae to obtain refraction and parallax and other needed parameters. In addition, because most of the navigation tables were designed for use at sea, they do not provide values for refraction at altitudes greater than about 2000 feet.

The calculations made from Lewis's celestial observations are given below in the following sequence despite the date or time in which they were made: 1) latitude from noon observation of the sun, 2) chronometer error at noon from Equal Altitudes observations, 3) chronometer time of an observation from any other observation for which the chronometer's time and sun's altitude is given, 4) longitude from Lunar Distance observations, and 5) magnetic declination.

LATITUDE CALCULATIONS

LATITUDE from MERIDIAN ALTITUDE of the SUN – JULY 28, 1805

At our encampment on Camp Island, near the junction of the three forks of the Missouri

Observed meridian altitude of the ⊙'s lower limb with octant by back observation	58°35'00"
Latitude deduced from this observation	45°24'54"

Calculations for Latitude from the Meridian Observation of the Sun – July 28, 1805

	58°35'	observed supplement of the double altitude of the sun's lower limb
-	04°23'20.6"	octant's index error in the back observation
	54°11'39.4"	observed angle, corrected for Index error
	180°00'00.0"	
-	54°11'39.4"	observed angle, corrected for Index error
	125°48'20.6"	apparent double altitude of sun's lower limb
÷	2	using the artificial horizon doubles the angle observed, divide by 2
	62°54'10"	apparent altitude of sun's lower limb = H
-	00°00'24"	refraction correction[1]
+	00°00'04"	parallax correction[2]
+	00°15'47"	sun's semidiameter[3]
	63°09'37"	altitude of sun's center with respect to earth's center
-	18°59'40"	sun's north declination at observation[4]
	44°09'57"	co-latitude
	90°00'00"	zenith
-	44°09'57"	co-latitude
	45°50'03"	latitude per this observation
	45°50'	latitude as octant can be read only to ½' (30") of arc
	45°55½	approximate actual latitude from Courses and Distances

Lewis's method (using correct index error and same corrections for refraction and parallax as above)

58°35'	observed complement of the double altitude of the sun's lower limb
÷ 2	
29°17'30"	observed angle halved
90°	
29°17'30"	
60°42'30"	altitude of sun's lower limb including index error, refraction and parallax
+2°11'40"	one half of the octant's index error[5]
62°54'10"	apparent altitude of the sun's lower limb = H
- 0°00'20"	refraction correction
+0°15'47"	sun's semidiameter
63°09'37"	
18°59'40"	sun's north declination
44°09'57"	co-latitude
90°	
45°50'03"	latitude per this observation

1. Lewis would have used Tables Requisite. The equation used here to calculate the correction for refraction is: [(983 x Bar Pressure in inches) ÷ (460 + Temp°F)] x cotangent H
2. Lewis likely used Tables Requisite. The equation used here to calculate the correction for the sun's parallax is: 8.794 x cosine H
3. Sun's noon semidiameter at 0° longitude: 25 July, 15'46.7"; 1 August, 15'47.5"
4. Sun's noon declination at 0° longitude: 28th, +19°04'00"; 29th, +18°50'01")
5. Lewis mistakenly used an index error of 2°40'; this accounts for most of the error in his calculations.

LATITUDE CALCULATIONS

LATITUDE from MERIDIAN ALTITUDE of the SUN – JULY 29, 1805

[At our encampment on Camp Island, near the junction of the three forks of the Missouri]	
Observed meridian altitude of the ☉'s lower limb with octant by back observation	59°07'
Latitude deduced from this observation	45°23'23.1"
Mean latitude from two meridian altitudes of ☉'s lower limb	45°24'08.5"

Calculations for Latitude from Meridian Observation of the Sun – July 29, 1805

	59°07'	observed supplement of the double altitude of sun's lower limb
-	04°23'20.6"	octant's index error in the back observation
	54°43'39.4"	observed angle, corrected for Index error
	180°00'00.0"	
-	54°43'39.4"	observed angle, corrected for Index error
	125°16'20.6"	apparent double altitude of sun's lower limb
÷	2	
	62°38'10"	apparent altitude of sun's lower limb = H
-	00°00'24"	refraction correction[1]
+	00°00'04"	parallax correction[2]
+	00°15'47"	sun's semidiameter[3]
	62°53'37"	altitude of sun's center with respect to earth's center
-	18°45'36"	sun's north declination at observation[4]
	44°08'01"	co-latitude
	90°00'00"	zenith
-	44°08'01"	co-latitude
	45°51'59"	latitude per this observation
	45°52'	latitude as octant can be read only to ½' (30") of arc
	45°55½'	approximate actual latitude from Courses and Distances

average of recalculated observations for 28 and 29 July 1805: 45°50'03" + 45°51'59" =	45°51'01"
average of recalculated observations for 28 and 29 July to nearest 30" =	45°51'
average latitude for 28 and 29 July 1805 as calculated by Lewis =	45°24'08.5

1. Lewis would have used Tables Requisite. The equation used here to calculate the correction for refraction is: [(983 x Bar Pressure in inches) ÷ (460 + Temp°F)] x cotangent H
2. Lewis would have used Tables Requisite. The equation used here to calculate the correction for the sun's parallax is: 8.794 x cosine H
3. Sun's noon semidiameter at 0° longitude: 25 July, 15'46.7"; 1 August. 15'47.5"
4. Sun's noon declination at 0° longitude: 29th, +18°50'01"; 30th, +18°35'44")

LOCAL TIME CALCULATIONS

CHRONOMETER ERROR at NOON from EQUAL ALTITUDES of the SUN – JULY 28, 1805 (page 1 of 2)

Observed equal altitudes of ☉ with sextant

	h m s		h m s		
A.M.	8 42 10	PM	4 21 46	accurate	
	8 43 42		4 23 21	doubtful	Altitude at the time of observation 72°08'15"
	8 45 15			lost by clouds	

Calculations for Chronometer Error from Equal Altitudes of the Sun – July 28, 1805

LL - C	C - UL	LL - C	C - UL	LL = lower limb, C = center, UL = upper limb
8:45:15	8:43:42	lost	4:23:21	
8:43:42	8:42:10	4:23:21	4:21:46	
0:01:33	0:01:32	?	0:01:35	4:21:46 + 1m33s = 4:23:19 center

Average chronometer time of the AM observation (center only)	08h43m42s
Average chronometer time of the PM observation (center only)	16h23m19s
Middle time = (AM time + PM time) ÷ 2 =	12h33m30.5s
½ Elapsed time = (PM time - AM time) ÷ 2 = 7h39m37s ÷ 2 =	03h49m48.5s
Latitude of Point of Observation; average of observations July 28 and 29, nearest 30"	45°51' N[1]
Sun's declination; 0° long. noon: 28th: 19°04'00"; 29th: 18°50'01"; at noon 111°[2]	+18°59'41"
Middle time of the observation by chronometer (center and center "corrected")	12h33m30.5s
Change in declination; daily: -13.98'; hourly: -34.96"; correction for change[3] =	+ 8.9s
Calculated chronometer time of Local Apparent Noon	12h33m39.4s
Local Apparent Time of Noon when sun is on the meridian	12h00m00.0s
Chronometer too fast on Local Apparent Time	33m39.4s
Local Apparent Time of Noon when sun is on the meridian	12h00m00.0s
Equation of Time at observation[4]	+ 6m04.1s
Local Mean Time of Solar Noon	12h06m04.1s
Chronometer time of Local Apparent Noon	12h33m39.4s
Chronometer too fast on Local Mean Time	27m35.3s

1. True latitude: 45°54'44"
2. True longitude: 111°30'40"
3. Correction for change in declination either from: Bowditch, Nathaniel, 1837 (reprinted 1864), The New American Practical Navigator, E & G. W. Blunt, New York, p. 219-220 or Ingram, E.L., 1911, Geodetic Surveying, McGraw Hill Book Co., p. 176-179. See following page.
4. In the 1805 Nautical Almanac, find the Equation of Time for July 28 (+6m04.4s) and July 29 (+6m03.3s) then either ratio the values to the Greenwich Apparent Time of the observation of 19h24m [12h + (111° ÷ 15)] or use Table VI in Tables Requisite.

LOCAL TIME CALCULATIONS

CHRONOMETER ERROR at NOON from EQUAL ALTITUDES of the SUN – JULY 28, 1805 (page 2 of 2)

Bowditch's Method for Correction for Sun's Changing Declination (modified to use without logs)

First part of the Correction =
180 ÷ [0.066667 x cotangent Latitude x sine (½ ET x 15) x (180 ÷ ET) x (180 ÷ Declin change per day)]
180 ÷ [0.066667 x cotangent 45°51' x sine (7h39m37s x 7.5) x (180 ÷ 7h39m37s) x (180 ÷ 13.98')]
180 ÷ [0.066667 x 0.970752 x 0.842942 x 23.497842 x 12.875536] = 16.504692 = 10.91s

Second part of the Correction =
180 ÷ [0.066667 x cotangent Declin x tangent (½ ET x 15) x (180 ÷ ET) x (180 ÷ Dec change per day)]
180 ÷ [0.066667 x cotangent 18°59'41" x tan (7h39m37s x 7.5) x (180 ÷ 7h39m37s) x (180 ÷ 13.98')]
180 ÷ [0.066667 x 2.905080 x 1.566792 x 23.4978420 x 12.875536] = 91.789903 = 1.96s

where ET = Elapsed Time

The first part of the correction is to be added to the Middle Time when the Sun is receding from the elevated pole (June 22-December 21 in the Northern Hemisphere) and subtracted from the Middle Time when it is advancing toward it (December 22 - June 21 in the Northern Hemisphere)... +10.9s

The second part of the correction is to be added to the Middle Time when the Declination is increasing (December 22 - June 21), but subtracted from the Middle Time when it is decreasing (June 22- December 21)... -2.0s

Net correction for changing declination = +10.9s - 2.0s = +8.9s

Ingram's Method – modified (correction to be subtracted from the middle time)

{[tan Lat ÷ sin (½ ET x 15)] - [(tan Dec ÷ tan (½ ET x 15)]} x [(Δ Dec ÷ 15) x ½ ET]

where: Lat = latitude
ET = elapsed time
Dec = sun's declination
Δ Dec = hourly change in sun's declination

1 {[tan 45°51' ÷ sine (3h49m48.5s x 15)] - [tan 18°59'41" ÷ tan (3h49m48.5s x 15)]} x [(-34.96 ÷ 15) x 3h49m48.5s]
2 [(1.030120 ÷ sine 57.452083) - (0.344225 ÷ tan 57.452083)] x (-2.330667 x 3h49m48.5s)
3 (1.030120 ÷ 0.843942) - (0.344225 ÷ 1.566792)
4 (1.222053 - 0.219700) x -8.926777
5 1.002353 x 9.926777 = -8.9 seconds (subtract a minus 8.9 sec = add 8.9 seconds)

LOCAL TIME CALCULATIONS

CHRONOMETER ERROR at NOON from EQUAL ALTITUDES of the SUN – JULY 29, 1805 (page 1 of 2)

Observed equal altitudes of the sun with sextant

	h m s		h m s	
A.M.	8 57 05.5	P.M.	4 05 50	
	8 58 41		4 07 24	Altitude by sextant at the time of observation 77°04'45"
	9 00 14		4 08 59	

Calculations for Chronometer Error from Equal Altitudes of the Sun – July 29, 1805

LL - C	C - LL	LL - C	C - UL
9:00:14	8:58:41	4:08:59	4:07:24
8:58:41	8:57:05.5	4:07:24	4:05:50
0:01:33	0:01:36.5	0:01:35	0:01:34

LL = lower limb, C = center, UL = upper limb

AM 9:00:14 - 8:57:05.5 = 3m08.5s; PM 4:08:59 - 4:05:50 = 3m09s

Average chronometer time of the AM observation (UL and LL only)	08h58m39.75s
Average chronometer time of the PM observation (UL and LL only)	16h07m24.5s
Middle time = (AM time + PM time) ÷ 2 =	12h33m02.13s
½ Elapsed time = (PM time - AM time) ÷ 2 = 7h08m44.75s = ÷ 2 =	03h34m22.37s
Latitude of the Point of Observation[1]; average of observation July 28th and 29th	45°51'N[1]
Sun's declination; 0° long. noon; 29th: 18°50'01"; 30th: 18°35'44"; at Noon 111°[2]	+18°45'37"
Middle time of the observation by chronometer	12h33m02.2s
Change in declination; daily: -14.28'; hourly: -35.71"; correction for change[3] =	+ 8.8s
Calculated chronometer time of Local Apparent Noon	12h33m11.0s
Local Apparent Time of Noon when sun is on the meridian	-12h00m00.0s
Chronometer too fast on Local Apparent Time	33m11.0s
Local Apparent Time of Noon when sun is on the meridian	12h00m00.0s
Equation of Time[4]	+ 6m02.8s
Local Mean Time of Solar Noon	12h06m02.8s
Chronometer time of Local Apparent Noon	12h33m11.0s
Chronometer too fast on Local Mean Time	27m08.2s

Date	Chronometer fast Local Apparent Time	Chronometer fast Local Mean Time
28	0h33m39.4s	0h27m35.3s
29	0h33m11.0s	0h27m08.2s
	0h00m28.4s	0h00m27.1s

the above data show that the chronometer was losing about 27 seconds per day on Local Mean Time = 1.125 seconds per hour. The calculations made for time of observation using latitude, sun's altitude (index error 8'45") and sun's declination, however, suggest that, between observations, the chronometer was losing about 2.5s/hr (60 seconds per day), see Local Time - Summary

1. True latitude: 45°55'44" N.
2. True longitude: 111°30'40" W
3. See footnote 3 for Local Time Calculations, 1805 July 28 and page 2 of 2 for that observation.
4. See footnote 4 for Local Time Calculations, 1805 July 28. Equation of Time at 0° longitude noon on 29 July: +6m03.3s; 30 July: +6m01.6s

LOCAL TIME CALCULATIONS

TIME of EQUAL ALTITUDES OBSERVATIONS
from LATITUDE and SUN'S DECLINATION and ALTITUDE
JULY 28, 1805 (page 1 of 1)

True Altitude of Sun's Center

8:43:42 - estimated 34m fast = 8:10 + 7:24 = 15:34; 16:23:21 - est. 33m30s fast = 15:48 + 7:24 = 23:12

AM		PM
72°08'15"	observed double altitude of sun's center	72°08'15"
00°08'45"	index error	00°08'45"
71°59'30"	double altitude corrected for index error	71°59'30"
÷ 2		÷ 2
35°59'45"	apparent altitude of center = H	35°59'45"
- 0°01'08"	refraction (see Latitude, footnote 1)	- 0°01'04"
+0°00'07"	parallax (see Latitude, footnote 2)	+0°00'07"
35°58'44"	altitude of sun's center per this observation	35°58'48"

Sun's declination at noon 0° longitude; 28th; 19°04'00"; declination 29th: 18°50'01"
Sun's declination at observation (111° W); AM = 19°01'55"; PM = 18°57'28"

Local Apparent Time of AM Observation – July 28, 1805

A	=	Latitude (average of Lewis's two Meridian Altitude observations) = 45°51' =	45.850000°
B	=	True altitude of sun's center (AM) = 35°58'44" =	35.978889°
C	=	(A + B) ÷ 2 =	40.914444°
D	=	B - C (absolute) =	04.935556°
E	=	½ polar distance (90° - declination) = ½ (90°- 19°01'55") = ½ 70.968056° =	35.484028°
F	=	$tangent^{-1}$ cotangent C x tangent D x cotangent E =	
	=	$tangent^{-1}$ 1.153844 x 0.086355 x 1.402775 = 0.139773 =	07.956877°
G	=	E ± F = 35.483960° - 7.957212°[1]	27.527151°
H	=	$cosine^{-1}$ tangent A x tangent G[2] = 1.030120 x 0.521169 = 0.536867 =	57.529390°
I	=	12 - (H ÷ 15) = 57.529390° ÷ 15 = Hour $Angle^{h}$ = 12 - 3.835293h = LAT of obs	08:09:53.0

Chronometer fast on Local Apparent Time: 8:43:42.0 - 8:09:53.0 = 33m49.0s
Chronometer fast on Local Mean Time: 8:43:42 - (8:09:53 + 6m04.2s) = 27m44.8s

Local Apparent Time of PM Observation – July 28, 1805

A	=	Latitude (average of Lewis's two Meridian Altitude observations) = 45°51'01" =	45.850000°
B	=	True altitude of sun's center (PM) = 35°58'48" =	35.980000°
C	=	(A + B) ÷ 2 =	40.915000°
D	=	B - C (absolute) =	04.935000°
E	=	½ polar distance (90° - declination) = ½ (90°- 18°57'28") = ½ 71.042222° =	35.521111°
F	=	$tangent^{-1}$ cotangent C x tangent D x cotangent E =	
	=	$tangent^{-1}$ 1.153821 x 0.086346 x 1.400856 = 0.139564 =	07.945090°
G	=	E ± F = 35.521111° - 7.945090°[1]	27.576022°
H	=	$cosine^{-1}$ tangent A x tangent G[2]	
		$cosine^{-1}$ 1.030120 x 0.522255 = 0.537985 =	57.453445°
I	=	12 + (H ÷ 15) = 57.453445° ÷ 15 = Hour $Angle^{h}$ = 12 + 3.830230h = LAT of obs	15:49:48.8

Chronometer fast on Local Apparent Time: 16:23:19.0 - 15:49:48.0 = 33m30.2s
Chronometer fast on Local Mean Time: 16:23:19 - (15:49:48 + 6m03.9s) 27m27.1s

Average of AM and PM, Local Apparent Time = 33m39.6s, by Equal Altitudes at noon = 33m39.4s
Average of AM and PM, Local Mean Time = 27m36.0s, by Equal Altitudes at noon = 27m35.3s

1. Subtract when A is greater than B, otherwise add.
2. Take the supplement to 180° when F is greater than E

LOCAL TIME CALCULATIONS

TIME of AM MAGNETIC OBSERVATION
from LATITUDE and SUN'S DECLINATION and ALTITUDE
JULY 29, 1805 (page 1 of 1)

⊙'s magnetic azimuth

Time by chronometer		Azimuth by circumferentor	Altitude of ⊙'s lower limb with sextant
	h m s	°	° ' "
A.M.	8 48 09	N 85 E	73 00 00
	8 53 57	N 86 E	74 58 15

Calculations for Magnetic Declination from AM Observations with the Sun – July 29, 1805

Recalculated latitude (observations of 1805 July 28 and 29) =	45°51'N
Average time by chronometer: 8:48:09 am + 8:53:57 am = 17:42:06 ÷ 2 =	8:51:03
Calculated Local Apparent Time = 8:51:03 - estimated 33m fast =	8:18
Greenwich Apparent Time = Calculated Local Apparent Time + 111° ÷ 15° = 7:24 =	15:42
Sun's declination; 29th: 18°50'01"; 30th: 18°35'44"; at 15:42 Greenwich App Time =	18°47'49"

73°59'07.5"	observed double altitude of sun by sextant
-0°08'45.0"	sextant's index error
73°50'22.5"	
÷ 2	
36°55'11"	apparent altitude = H
- 0°01'05"	refraction (see Latitude, footnote 1)
+0°00'07"	parallax (see Latitude, footnote 2)
+0°15'47"	sun's semidiameter (Jul 25: 15'46.7"; Aug 1: 15'47.5", at obs at 111° = 15'47.2")
37°10'00"	altitude of sun's center per this observation

Time of Magnetic Declination Observation from Declination, Altitude and Latitude

A	=	Latitude	45.850000°	
B	=	True altitude sun's center	37.166667°	(37°10'00")
C	=	(A + B) ÷ 2	41.508333°	
D	=	B - C (absolute)	04.341667°	
E	=	½ polar distance	35.601528°	½ of (90° - 18°47'49")
F	=	$\tan^{-1}$ cot C x tan D x cot E	06.832705°	
G	=	E ± F (- if A > B)	28.768822°	
H	=	$\cos^{-1}$ tan A x tan G	55.557191°	
I	=	H ÷ 15 = Hour angle	03.703813h =	3:42:13.7
LAT		= 12 - I = 12 - 3.703813 =	08.296187h =	8:17:46.3 Local Apparent Time of observation ave
Chronometer time (average)			08.850833h =	8:51:03.0
Chronmeter fast LAT at observation			00.554328h =	0:33:16.7 seconds
Chronometer fast LMT at observation =			8:51:03 - (8:17:46.3 + 6m04.2s) = 27m12.5s	

LOCAL TIME CALCULATIONS

TIME of AM EQUAL ALTITUDES OBSERVATION
from LATITUDE and SUN'S DECLINATION and ALTITUDE
JULY 29, 1805 (page 1 of 1)

Observed equal altitudes of the sun with sextant

	h m s		h m s	
A.M.	8 57 05.5	P.M.	4 05 50	
	8 58 41		4 07 24	Altitude by sextant at the time of observation 77°04'45"
	9 00 14		4 08 59	

True Altitude of Sun's Center

AM	
77°04'45"	altitude by sextant at the time of observation
-0°08'45"	sextant's index error
76°56'00"	
÷ 2	
38°28'00"	H
- 0°01'01"	refraction (see Latitude, footnote 1)
+0°00'07"	parallax (see Latitude, footnote 2)
38°27'06"	altitude of sun's center per this observation

Recalculated latitude (observations of 1805 July 28 and 29) =	45°51'N
Average time by chronometer AM =	8:58:39.75
Calculated Local Apparent Time = 8:58:40 - estimated 33m fast =	8:26
Greenwich Apparent Time = Calculated Local Apparent Time + 111° ÷ 15° = 7:24 =	15:50
Sun's declination; Jul 29: 18°50'01"; Jul 30: 18°35'44"; at 15:50 Greenwich App Time =	18°47'44"

Time of AM Observation from Declination, Altitude and Latitude

A	=	Latitude	45.850000°	
B	=	True altitude sun's center	38.451667°	(38°27'06")
C	=	(A + B) ÷ 2	42.150833°	
D	=	B - C (absolute)	03.699167°	
E	=	½ polar distance	35.602222°	½ of (90° - 18°47'44")
F	=	$\tan^{-1}$ cot C x tan D x cot E	05.696807°	
G	=	E ± F (- if A > B)	29.905415°	
H	=	$\cos^{-1}$ tan A x tan G	53.667188°	
I	=	H ÷ 15 = Hour angle	03.577813h =	03:34:40.1
LAT		= 12 = I = 12 - 3.577813 =	08.422187h =	08:25:19.9 Local Apparent Time of obs ave
Chronometer time (average)			08.977824h =	08:58:40.2
Chronometer fast LAT at observation			00.555497h =	00:33:20.3 seconds

Chronometer fast LMT at observation = 8:58:40.2 - (8:25:19.9 + 6m03.0s) = 27m17.3s

Using an index error of 8' makes the chronometer 33m18.0s fast on LAT for this observation
Using an index error of 7' makes the chronometer 33m14.9s fast on LAT for this observation

LOCAL TIME CALCULATIONS

TIME of PM EQUAL ALTITUDES OBSERVATION
from LATITUDE and SUN'S DECLINATION and ALTITUDE
JULY 29 1805 (page 1 of 1)

Observed equal altitudes of the sun with sextant

	h m s		h m s	
A.M.	8 57 05.5	P.M.	4 05 50	
	8 58 41		4 07 24	Altitude by sextant at the time of observation 77°04'45"
	9 00 14		4 08 59	

True Altitude of Sun's Center

PM	
77°04'45"	altitude by sextant at the time of observation
-0°08'45"	sextant's index error
76°56'00"	
÷ 2	
38°28'00"	
- 0°00'59"	refraction (see Latitude, footnote 1)
+0°00'07"	parallax (see Latitude, footnote 2)
38°27'08"	altitude of sun's center per this observation

Recalculated latitude (observations of 1805 July 28 and 29) =	45°51'N
Average time by chronometer PM =	16:07:24.5
Calculated Local Apparent Time = 16:07:24.5 - estimated 33m fast =	15:34
Greenwich Apparent Time = Calculated Local Apparent Time + 111° ÷ 15° = 7:24 =	22:58
Sun's declination; Jul 29: 18°50'01"; Jul 30: 18°35'44"; at 22:58 Greenwich App Time =	18°43'29"

Time of PM Observation from Declination, Altitude and Latitude

A	=	Latitude	45.850000°	
B	=	True altitude sun's center	38.452222°	(38°27'08")
C	=	(A + B) ÷ 2	42.151111°	
D	=	B - C (absolute)	03.698889°	
E	=	½ polar distance	35.637639°	½ of (90° - 18°43'29")
F	=	$\tan^{-1}$ cot C x tan D x cot E	05.688942°	
G	=	E ± F (- if A > B)	29.948697°	
H	=	$\cos^{-1}$ tan A x tan G	53.593468°	
I	=	H ÷ 15 = Hour angle	03.572898h =	03:34:22.4
LAT		= 12 = I = 12 + 3.572898 =	15.572898h =	15:34:22.4 Local Apparent Time of obs ave
Chronometer time (average)			16.123426h =	16:07:24.3
Chronometer fast LAT at observation			00.550531h =	00:33:01.9 seconds

Chronometer fast LMT at observation = 16:07:24.3 - (15:34:22.4 + 6m02.5s) = 26m59.4s

LOCAL TIME CALCULATIONS

TIME (AVERAGE) of PM MAGNETIC DECLINATION OBSERVATIONS
from LATITUDE and SUN'S DECLINATION and ALTITUDE
JULY 29, 1805 (page 1 of 1)

	Time	Azimuth	Altitude (lower limb)
P.M.	5 07 47	S 72 W	55 44 30
	5 13 04	S 73 W	53 52 45

Altitude of the Sun's Center

54°48'37.5"	average double altitude of sun's lower limb + index error
-0°08'45"	sextant's index error
54°39'52.5"	double altitude
÷ 2	
27°19'56"	apparent altitude = H
- 0°01'30"	refraction (see Latitude, footnote 1)
+0°00'08"	parallax (see Latitude, footnote 2)
+0°15'47"	semidiameter; 0° longitude at noon Jul 25: 15'46.7"; Aug 1: 15'47.5"
27°34'21"	altitude of sun's center per this observation

Recalculated latitude (observations of 1805 July 28 and 29) =	45°51'N
Average time by chronometer PM =	17:10:25.5
Calculated Local Apparent Time = 17:10:25.5 - estimated 33m fast =	16:37
Greenwich Apparent Time = Calculated Local Apparent Time + 111° ÷ 15° = 7:24 =	24:01
Sun's declination; Jul 29: 18°50'01"; Jul 30: 18°35'44"; at 22:58 Greenwich App Time =	18°42'52"

Calculated Local Apparent Time of PM Magnetic Declination Observation

A	=	Latitude (average of Lewis's two Meridian Altitude observations) = 45°51' =	45.850000°
B	=	True altitude of sun's center (average) = 27°34'21"	27.572500°
C	=	(A + B) ÷ 2 =	36.711250°
D	=	B - C (absolute) =	09.138750°
E	=	½ polar distance (90° - declination) = ½ (90°- 18°42'52") =	35.642778°
F	=	tangent^{-1} cotangent C x tangent D x cotangent E =	
	=	tangent^{-1} 1.341053 x 0.160868 x 1.394584 = 0.300857 =	16.744274°
G	=	E ± F = 35.642778° - 16.744274°[1] =	18.898504°
H	=	cosine^{-1} tangent A x tangent G[2]	
		cosine^{-1} 1.030120 x 0.342347 = 0.352659 =	69.349978°
I	=	12 + (H ÷ 15) = 69.349978° ÷ 15 = Hour Angle^h = 12 + 4.623332 = LAT of obs	16:37:24.0
Chronometer too fast on Local Apparent Time = 17:10:25.5 - 16:37:24.0 =			33m01.5s
Chronometer too fast on Local Mean Time - 17:10:25.5 - (16:37:24 + 6m02.4s) =			26m59.0s

1. Subtract when A is greater than B, otherwise add.
2. Take the supplement to 180° when F is greater than E.

LOCAL TIME CALCULATIONS

SUMMARY of OBSERVATIONS TAKEN and TIMES, THREE FORKS of the MISSOURI
JULY 28-29, 1805 (page 1 of 1)

Day	Obs	Chrono Time	Chrono fast on LAT	True LAT of Obs	GAT of Obs +7:24	Equation of Time	GMT of Obs
28	=Alt⊙AM	08:43:42.0	00:33:49.0	08:09:53.0	15:33:53.0	+06m04.2s	15:39:57.2
28	⊙ noon	12:33:39.4	00:33:39.4	12:00:00.0	19:24:00.0	+06m04.1s	19:30:04.1
28	=Alt⊙PM	16:23:19.0	00:33:30.2	15:49:48.8	23:13:48.8	+06m03.9s	23:19:52.6
29	⊙ Magav	08:51:03.0	00:33:16.7	08:17:46.3	15:41:46.3	+06m03.0s	15:47:49.3
29	=Alt⊙AM	08:58:40.2	00:33:20.3	08:25:19.9	15:49:19.9	+06m03.0s	15:55:22.9
29	⊙ noon	12:33:11.0	00:33:11.0	12:00:00.0	19:24:00.0	+06m02.8s	19:30:02.8
29	=Alt⊙PM	16:07:24.3	00:33:01.9	15:34:22.4	22:58:22.4	+06m02.5s	23:04:24.9
29	☽-⊙1	16:23:12.6	00:33:06.4[1]	15:50:06.2	23:14:06.2	+06m02.5s	23:20:08.7
29	☽-⊙1	16:23:12.6	00:33:01.9[2]	15:50:10.7	23:14:10.7	+06m02.5s	23:20:13.2
29	☽-⊙2	16:49:09.6	00:33:05.9[1]	16:16:03.7	23:40:03.7	+06m02.5s	23:46:06.2
29	☽-⊙2	16:49:09.6	00:33:00.9[2]	16:16:08.7	23:40:08.7	+06m02.5s	23:46:11.2
29	⊙ Magav	17:10:25.5	00:33:01.5	16:37:24.0	24:01:24.0	+06m02.5s	00:07:26.5 30th
29	Antares-☾	20:57:26.8	00:33:00.9[1]	20:24:25.9	27:48:25.9	+06m02.2s	03:54:28.1 30th
29	Antares-☾	20:57:26.8	00:32:51.3[2]	20:24:35.5	27:48:35.5	+06m02.2s	03:54:37.7 30th
29	Polaris	21:27:00	00:33:00.4[1]	20:53:59.6	28:17:59.6	+06m02.2s	04:24:01.8 30th
29	Polaris	21:27:00	00:32:49.8[2]	20:54:10.1	28:18:10.1	+06m02.2s	04:24:12.3 30th

Note: values in red were determined for noon from the Equal Altitudes observations for those days
Note: values in blue were derived from Patterson's Form III for finding the time of observation given the latitude, sun's altitude and sun's declination.

The trend from the AM and PM Equal Altitudes calculations (only) each day appears to show a loss of 2.46 seconds per hour = 59 seconds per 24h, yet the loss between noon 28th and noon 29th is only half that = 28.5 seconds. This difference may result from a change in the sextant's index error from +8'45" to some other value (need to check) or from improper winding of the chronometer. Lewis (22 July 1804) indicates the chronometer was wound at noon, but did he change that to evening after leaving Fort Mandan, inasmuch as the chronometer appears to have lost only 10 seconds between the PM Equal altitudes observation of 28 July and the AM Equal altitudes of 29 July per calculated Local Apparent Time of observation.

Obs = Observation
Chrono = Chronometer
LAT = Local Apparent Time
GAT = Greenwich Apparent Time
GMT = Greenwich Mean Time
= Alt = Equal Altitudes of the Sun
⊙ = Sun
Mag = Magnetic Declination with Sun
☽ = Moon (Lunar Distance)
Polaris = Magnetic Declination with Polaris

1. Calculated from chronometer's noon error 28th and 29th from the Equal Altitudes observations then projecting by linear regression using the error on Local Mean Time and converting back to LAT.

2. Calculated by linear regression from 1) the calculated Local Apparent Time of AM and PM observations of Equal Altitudes on the 29th and the noon error on the 29th, 2) the same as in (1) above plus the calculated LAT of the AM and PM observations for Magnetic declination. Then both (1) and (2) were averaged.

LONGITUDE CALCULATIONS

LONGITUDE from LUNAR DISTANCE from THE SUN – FIRST SET of OBSERVATIONS
July 29, 1805 (page 1 of 5)

Observed time and Distance of ☉'s and ☽'s nearest Limbs with Sextant. ☉ West

	Time	Distance	Time	Distance
	h m s	° ' "	h m s	° ' "
P.M.	4 14 42	49 43 30	4 23 12	49 46 30
	4 17 24	49 44 00	4 24 14	49 46 45
	4 19 34	49 44 45	4 25 18	49 47 00
	4 21 12	49 45 00	4 26 26	49 47 15
	4 22 09	49 45 54[2]	4 27 24	49 47 30

1. This should have been 49°45'52.5" because the sextant could be read only to the nearest 7½"

Calculations for Longitude from Lunar Distance from the Sun, first set of observations – July 29, 1805

Average time by chronometer time: 4:22:09.5; average separation by sextant: 49°45'48.75". A plot of the data, however, suggests using 4:23:12.6 p.m. and 49°46'12.2" as averaged from data sets 2, 3, and 5 through10.

True Time of Sun-Moon Observation No. 1 (see LOCAL TIME CALCULATIONS – Summary)

Ave chrono	fast on LAT	True LAT	111°W	G App Time	Eq of Time	G Mean Time
16:23:12.6	00:33:06.4	15:50:06.2	+7h24m	23:14:06.2	+06m02.5s	23:20:08.7

Sun - Moon Data from Nautical Almanac Calculated for average time of first set of observations

RA Sun 29th	8h32m49.6s	Dec Sun 29th	+18°50'01" N	SD Sun 25th	15'46.7"
RA Sun 30th	8h36m44.5s	Dec Sun 30th	+18°35'44" N	SD Sun 32nd	15'47.5"
RA Sun obs	8h34m39.6s	Dec Sun obs	+18°43'20" N	SD Sun obs	15'47.2"

RA Moon 29th 12h	168°36'	= 11h14m24s	Dec Moon 29th 12h	0°12' S
RA Moon 29th 24h	174°48'	= 11h29m12s	Dec Moon 29th 24h	3°08' S
RA Moon at obs	174°24'	= 11h37m36s	Dec Moon at obs	-2°56'47"

SD Moon 29th 12h	15'59"	HP Moon 29th 12h	58'40"	Eq of Time 29th +6:03.3
SD Moon 29th 24h	15'52"	HP Moon 29th 24h	58'12"	Eq of Time 30th +6:01.6
SD Moon at obs	15'52"	HP Moon at obs	58'14"	Eq of Time obs +6:02.5

RA = right ascension
Dec = declination
SD = semidiameter
HP = horizontal parallax
Eq = equation
obs = observation

LONGITUDE CALCULATIONS

LONGITUDE from LUNAR DISTANCE from the SUN – FIRST SET of OBSERVATIONS
JULY 29, 1805 (page 2 OF 5)

True Altitude of the Sun's center at the time of Sun-Moon, first set of observations – July 29, 1805

A	=	Latitude: average from meridian observations July 28 and 29 (to nearest 30") =	45°51'00"
B	=	Sun's Declination at the time of the observation	18°43'20"
C	=	Sun's Hour Angle at the time of the observation 3:50:06.2 x 15 =	57°31'33"
D	=	tangent^{-1} of tangent B x secant C =	
	=	tangent^{-1} of (0.338914 x 1.862477) = 0.631219 =	32.260900°
E[1]	=	A ± D =	13.589100°
F	=	sine^{-1} of (sine B x cosecant D x cosine E)	
	=	sine^{-1} of (0.320980 x 1.873447 x 0.972006) = 0.584506 = True Alt (Hc) =	35.768073°
			35°46'05"

1. Add if Declination and Latitude are of different signs or C is greater than 90°)

Apparent ("observed") Altitude of the Sun's Center

G_1	=	Refraction = [(983 x inches Hg ÷ (460 + t°F) x cotangent Hc (1st trial), then Ha afterwards	
	=	{[(983 x 25.79) ÷ (460 + 70°]} ÷ 3600} x cot Hc, then Ha; G_1 = 0.013287 x cot Hc, Ha	
G_2	=	Parallax = [(8.794 ÷ 1.015 AU) ÷ 3600] x cos Hc (1st trial), then Ha afterwards =	
		(8.664 ÷ 3600) x cos Hc, then Ha; G_2 = 0.002407 x cos Hc, Ha	
H	=	1) 35.768073° + (cot 35.768073° x 0.013287) - (cos 35.768073° x 0.002407) =	35.784565° (Ha1)
		2) 35.784565° - (cot 35.784565° x 0.013287) + (cos 35.784565° x 0.002407) =	35.768084°
		3) 35.768084° - 35.768073°(Hc) = +0.000011°	
		4) 35.784565° - 0.000011° = 35.784554° = Ha2	
		5) 35.784554° - (cot 35.784554° x 0.01327) + (cos 35.784554° x 0.002407) =	35.768073° = Hc
		Therefore the sun's apparent altitude (Ha) =	35.784554°
		Δh☽ = 35.784554° - 35.768073° = 0.016481°	35°47'04"

Moon's Hour Angle at Time of Observation

1.	Sun's Right Ascension at the time of the observation	08h34m39.6s
2.	Local Apparent Time of the observation, pm =	03h50m06.2s
3.	Sum = Right Ascension of the Meridian at time of observation	12h24m45.8s
4.	Moon's Right Ascension at time of the observation =	11h37m36.0s
5.	Moon's Hour Angle in Time	00h47m09.8s

True Altitude of the Moon's Center

A	=	Latitude: from meridian observations July 28 and 29 (rounded to nearest 30") =	45°51'00"
B	=	Moon's Declination at the observation	-02°57'
C	=	Moon's Hour Angle as an angle = 47m09.8s (use 47m10s) x 15 =	11°47'30"
D	=	cotangent^{-1} (cotangent B x cosine C)	
	=	cotangent^{-1} (-19.405133 x 0.978897) = -18.995629 =	-3°00'48.5"
E[1]	=	A ± D	
	=	45°51' - -3°08'48.5" =	48°51'48.5"
F	=	cosecant^{-1} of (cosecant B x sine D x secant E)	
	=	cosecant^{-1} of (-19.430882 x -0.052571 x 1.520091) = -1.552771 = True alt (Hc)	40.091486°
			40°05'29"

1. Add if Declination and Latitude are of different signs or C is greater than 90°

LONGITUDE CALCULATIONS

LONGITUDE from LUNAR DISTANCE from the SUN – FIRST SET of OBSERVATIONS
JULY 29, 1805 (page 3 of 5)

Apparent ("observed") Altitude of Moon's Center

G_1 = Refraction = 0.013287 x cotangent Hc , Ha (see Sun, above)
G_2 = Parallax = moon's Horizontal Parallax (0.970496) x cosine Hc, Ha
H = 1) 40.091486° + (cot 40.091486° x 0.013287) - (cos 40.091486° x 0.970496) = 39.364824° (Ha1)
2) 39.364824° - (cot 39.364824° x 0.013287) + (cos 39.364824 x 0.970496) = 40.098940°
3) 40.098940° - 40.091486°(Hc) = 0.007454°
4) 39.364824° - 0.007454° = 39.357369° = Ha2
5) 39.357369° - (cot 39.357369° x 0.013287) + (cos 39.357369° x 0.970496) = 40.091562°
6) 40.091562 - 40.091486°(Hc) = 0.000076°
7) 39.357369° - 0.000076° = 39.357293° = Ha3
8) 39.357293° - (cot 39.357293° x 0.013287) + (cos 39.357293° x 0.970496) = 40.091486°= Hc
Therefore the moon's apparent altitude (Ha) = 39.357293°
Δh☽ = 39.357293° - 40.091486° = - 0.734193° 39°21'26"

Moon's Augmentation, Sun-Moon Observation No. 1

Augmentation = (sine Ha x ☽'s SD) ÷ 60 = (sine 39°21'26" x 15'52") ÷ 60 = 0.002795° = 10.1"

Apparent Angular Distance of Sun and Moon

(Sextant distance - index error + sun's semidiameter + moon's semidiameter + moon's augmentation)

49°46'12"	average distance by sextant
- 0°08'45"	sextant's index error
49°37'27"	average distance corrected for index error
+0°15'47"	sun's semidiameter
+0°15'52"	moon's semidiameter
+0°00'10	moon's augmentation
50°09'16"	apparent sun-moon separation; 50.183875 at IE 7'; 50.175531 at 7'30"; 50.167198 at 8'

LONGITUDE from LUNAR DISTANCE from the SUN – FIRST SET of OBSERVATIONS
JULY 29, 1805 (page 4 of 5)

True separation of the Sun and the Moon, first set of observations – July 29, 1805

Patterson, Form V (modified)

A	=	Apparent separation of moon's center from sun's center	50°09'16"
B	=	Apparent altitude of moon's center	39°21'26"
C	=	Apparent altitude of sun's center	35°47'04"
D	=	½ (B + C) =	37°34'15"
E[1]	=	C ~ D =	01°47'11"
F	=	A ÷ 2 =	25°04'38"
G	=	tangent-1 of (tangent D x cotangent E x tangent F) =	
		tangent-1 of (0.769293 x 32.063126 x 0.467950) =	
		tangent-1 of 11.542415	85°02'54"
H[2]	=	F ± G = 25°04'38" - 85°02'54" =	59°58'16"
I[3]	=	F ± G = 25°04'38" + 85°02'54" =	110°07'32"
K	=	180 ÷ moon's horizontal parallax = 180 ÷ 58'14" = (58.23') =	3°05'28"
L	=	180 ÷ (cosecant B x tangent I x K) =	
		180 ÷ [(1.576906 x -2.736594 x 3°05'28") = 13.339227] = -13.494035 =	00°13'30"
M	=	refraction of I = ref (180°- I) 70° = (from tables)	00°00'18"
N	=	L-M = 13'30" - 18" = 1st correction =	00°13'12"
O[4]	=	A ± N = 50°09'16" - 13'12" =	49°56'04"
P	=	refraction and parallax of H for sun =	
	=	r of 60° (from tables)=	00°00'27"
Q[5]	=	O ± P = 49°56'04" + 27" =	49°56'31"
R[6]	=	correction from Table XIII (Tables Requisite)	00°00'04"
S[7]	=	true distance = Q ± R = 49°56'31" + 4" =	49°56'35"
T[8]	=	preceding distance in Nautical Almanac (21h) =	48°45'25"
U[8]	=	following time in Nautical Almanac (24h) =	50°20'29"

Calculating the Longitude

48°45'25" x_1; 21h y_1			
	49°56'35" >	23h14m44.8s	
50°20'29" x_2, 24h y_2,		15h50m06.2s	average Local Apparent Time of observation
		07h24m38.6s	difference in time
		x 15	
		111°09'39" W	111°10' W (should be 111°30'40")

---

1. The symbol ~ means "absolute value".
2. Add (+) if C is greater than B, if not subtract (-); therefore F - G.
3. Subtract (-) if C is greater than B, if not add (+); therefore F + G.
4. Subtract (-) if H or I is greater than 90°; or H is greater than I; therefore A - N.
5. Add (+) if H or I is greater than 90°; or H is less than I; therefore O + P
6. In Table VII find the correction of moon's altitude, then in Table XIII under the nearest degree to Q at the top, find two numbers; one opposite to the nearest minute to moon's correction of altitude found as above, and the other opposite the nearest minute to first correction (N) and the difference of these two numbers will be the third correction. This correction may, without sensible error, be generally omitted.
7. Add (+) if Q is less than 90°, if not, subtract (-) = Q + R
8. T and U; These are to be found in Nautical Almanac from page 8th to page11th of the month opposite the day of the month and the sun or star from which the moon's distance was observed; taking out the two distances, which are next greater and next less than the true distance (S) calling that the preceding distance which comes first in the order of time and the other the following distance)

LONGITUDE from LUNAR DISTANCE from the SUN – FIRST SET of OBSERVATIONS JULY 29, 1805 (page 5 of 5)

True separation of the Sun and the Moon, first set of observations – July 29, 1805

Jean Borda's Method (1787), modified to use RPN calculator

A	= ☽'s Apparent Altitude =	39°21'26"	m
B	= ☉'s Apparent Altitude =	35°47'04"	s
C	= Apparent Distance =	50°09'16"	d
D	= ☽'s True alt =	40°05'29"	M
E	= ☉'s True alt =	35°46'05"	S
F	= (A + B + C) ÷ 2 =	62°38'53"	(m + s + d) ÷ 2
G	= (A + B - C) ÷ 2 =	12°29'37"	(m + s - d) ÷ 2
H	= (D + E) ÷ 2 =	37°55'47"	(M + S) ÷ 2

I = cosine^{-1} √‾ secant A x secant B x cosine D x cosine E x cosine F x cosine G
= cosine^{-1} √‾ 1.293314 x 1.232707 x 0.765017 x 0.811390 x 0.459455 x 0.976320 =
= cosine^{-1} √‾ 0.443915 = cosine^{-1} of 0.666270 = 48°13'13"

J = sine (I + H) = sine 86°09'00" = 0.997668
K = sine (I - H) = sine 10°17'25.6" = 0.178638
L = sine^{-1} √‾ (J x K) = sine^{-1} √‾ 0.178221 = sine^{-1} of 0.422163 = 24.971197° = 24°58'16"

True Distance = 2L = 2 x 24°58'20" = 49.942393° = 49°56'33"

48°45'25" x_1, 21h y_1;		
49°56'33" >	23h14m41s	Greenwich Apparent Time
50°20'29" x_2, 24h y_2	15h50m06s	average Local Apparent Time of observation 07s
	07h24m35s	difference in time
	x 15	degrees of longitude per hour
	111°08'45"	111°09' west longitude (should be 111°30'40")

Note: using a Local Apparent Time of 15h50m10.7s (see Local Time - Summary) yields a longitude of 111°07'37" W.

LONGITUDE from LUNAR DISTANCE from the SUN – SECOND SET of OBSERVATIONS JULY 29, 1805 (page 1 of 5)

Observed time and Distance of ☉'s and ☽'s nearest Limbs with Sextant. ☉ West

	Time	Distance	Time	Distance
	h m s	° ' "	h m s	° ' "
P.M.	4 45 25	49 54 00	4 50 44	49 56 45
	4 46 37	49 54 45	4 51 36	49 57 15
	4 47 40	49 55 15	4 52 36	49 57 45
	4 48 52	49 55 45	4 53 37	49 58 00
	4 49 47	49 56 15	4 54 36	49 58 15

Calculations for Longitude from Lunar Distance from the Sun, second set of observations – July 29, 1805

Average time by chronometer: 4:50:09.0; average separation by sextant: 49°56'24.0".
A plot of the data suggests 4:49:09.6 and 49°55'58.1" (values 1 - 8 only),

True Time of Sun-Moon Observation No. 2 (see LOCAL TIME CALCULATIONS – Summary)

ave chrono	fast LAT	true LAT	111°W	GAT	Eq of Time	GMT
16:49:09.6	00:33:05.9	16:16:03.7	+7h24m	23:40:03.7	+06m02.5s	23:46:06.2

Sun - Moon Data from Nautical Almanac calculated for average time of second set of observations

RA Sun 29th	08h32m49.6s	Dec Sun 29th	+18°50'01"	SD Sun 25th	15'46.7"
RA Sun 30th	08h36m44.5s	Dec Sun 30th	+18°35'44"	SD Sun 32nd	15'47.5"
RA Sun obs2	08h34m43.8s	Dec Sun obs2	+18°43'04"	SD Sun obs2	15'47.2"

RA Moon 29th 12h	168°36'	= 11h14m24s	Dec Moon 29th 12h	0°12' S
RA Moon 29th 24h	174°48'	= 11h39m12s	Dec Moon 29th 24h	3°08' S
RA Moon at obs2	174°37'42"	= 11h38m31s	Dec Moon at obs2	-3°03'

SD Moon 29th 12h	15'59"	HP Moon 29th 12h	58'40"	Eq of Time 29th	+6:03.3
SD Moon 29th 24h	15'52"	HP Moon 29th 24h	58'12"	Eq of Time 30th	+6:01.6
SD Moon at obs 2	15'52.2"	HP Moon at obs 2	58'12.8"	Eq of Time at obs	+6:02.5

RA = right ascension
Dec = declination
SD = semidiameter
HP = horizontal parallax
Eq = equation
obs = observation

True Altitude of the Sun's Center, second set of Sun-Moon observations

A	=	Latitude: average of meridian observations July 28 and 29, to nearest 30" =	45°51'00"
B	=	Sun's Declination at the time of the observation	18°43'04"
C	=	Sun's Hour Angle at the time of observation = 4:16:03.7[1] pm x 15 =	64°00'55.5"
D	=	tangent-1 of (tangent B x secant C) =	
	=	tangent-1 of (0.338827 x 2.282431) = 0.773350 =	37°43'00"
E[1]	=	A ± D = 45°51' - 37°43' =	8°08'
F	=	sine-1 of (sine B x cosecant D x cosine E) =	
	=	sine-1 of (0.320907 x 1.634636 x 0.989942) = 0.519289 = True Altitude (Hc) =	31.284601°
			31°17'05"

1. Add (+) if A and B are of different signs or C is greater than 90°, otherwise subtract (-).

LONGITUDE CALCULATIONS

LONGITUDE from LUNAR DISTANCE from the SUN – SECOND SET of OBSERVATIONS
JULY 29, 1805 (page 2 of 5)

Apparent (“observed”) Altitude of the Sun’s Center

G_1	=	Refraction (R) = [(983 x inches Hg ÷ (460 + t°F) x = cotangent Hc (1st trial), then Ha afterwards	
	=	{[(983 x 25.79) ÷ (460 + 80°]} ÷ 3600} x cot Hc, then Ha; G_1 = 0.013041° x cot Hc, Ha	
G_2	=	Parallax (P) = [(8.794 ÷ 1.015 AU) ÷ 3600] x cos Hc (1st trial), then Ha afterwards	
		(8.664 ÷ 3600) x cos Hc, Ha; G_2 = 0.002407° x Hc, Ha	
H	=	1) 31.284601° + (cot 31.284601° x 0.013041) - (cos 31.284601° x 0.002407) =	31.304006° (Ha1)
		2) 31.304006° - (cot 31.304006° x 0.013041) + (cos 31.304006° x 0.002407) =	31.284617°
		3) 31.284617° - 31.284601°(Hc) = 0.000016°	
		4) 31.304006° - 0.000016° = 31.303990° = Ha2	
		5) 31.303990° - (cot 31.303990° x 0.013041) + (cos 31.303990° x 0.002407) =	31.284601° = Hc
		Therefore sun’s apparent altitude (Ha) =	31.303990°
		Δh☽ = 31.3039990° - 31.284601° = 0.019389°	31°18'14"

Moon’s Hour Angle at Time of Observation

1.	Sun’s Right Ascension at time of the observation =	08h34m43.8s
2.	Local Apparent Time of the observation, pm =	04h16m03.7s
3.	Sum	12h50m47.5s
4.	Moon’s Right Ascension at time of the observation	11h38m31.0s
5.	Moon’s Hour Angle as Hour	01h12m16.5s

True Altitude of the Moon’s Center for Sun-Moon Observation No. 2

A	=	Latitude: average of meridian observations July 28 & 29, to nearest 30" = 45°51'	45.850000
B	=	Moon’s Declination at the time of the observation = -3°03' =	-3.500000
C	=	Moon’s Hour Angle as Angle = 1h12m16.5s x 15 = 18°04'07.5" =	18.068750
D	=	cotangent-1 of (cotangent B x cosine C)	
	=	cotangent-1 of (-18.767754 x 0.950685) = -17.842223 = -3.207891 =	-3°12'28"
E[1]	=	A ± D = 45°51' - -3°12'28" = 49.057891 =	49°03'28"
F	=	cosecant-1 of (cosecant B x sine D x secant E)	
	=	cosecant-1 of (-18.794377 x -0.055961 x 1.526024) = -1.604946 = True Alt (Hc)	38.540971° 38°32'27.5"

1. Add (+) if A and B are of different signs or C is greater than 90°, otherwise subtract (-).

Apparent (“observed”) Altitude of Moon’s Center

G_1	=	Refraction (R) = 0.013041 x cotangent Hc, Ha (see Sun, above)	
G_2	=	Parallax (P) = moon’s Horizontal Parallax (0.970215) x cosine Hc, Ha	
H	=	1) 38.540971° + (cot 38.540971° x 0.013041) - (cos 38.540971° x 0.970215) =	37.798476° (Ha1)
		2) 37.798476° - (cot 37.798476° x 0.013041) + (cos 37.798476° x 0.970215) =	38.548298°
		3) 38.548298° - 38.540971°(Hc) = 0.007327°	
		4) 37.798476° - 0.007327° = 37.791148° = Ha2	
		5) 37.791148° - (cot 37.791148° x 0.013041) + (cos 37.791148° x 0.970215)° =	38.541043°
		6) 38.541043° - 38.540971°(Hc) = 0.000072°	
		7) 37.791148° - 0.000076° = 37.791077° = Ha3	
		8) 37.791077° - (cot 37.791077° x 0.013041) + (cos 37.791077° x 0.970215)° =	38.540971° = Hc
		Therefore moon’s apparent altitude (Ha) =	37.791077°
		Δh☽ = 37.791077° - 38.540971° = - 0.749894°	37°47'28"

Moon’s Augmentation from its Apparent Altitude in Sun-Moon Observation No. 2

Augmentation° = (sine Ha x ☽’s SD) = (sine 37°47'28" x 15'52"°) ÷ 60 = 0.002701° = 9.7"

LONGITUDE from LUNAR DISTANCE from the SUN – SECOND SET of OBSERVATIONS
JULY 29, 1805 (page 3 of 5)

Correct observed distance for Observation No. 2 for index error, semidiameters and augmentation
Observed distance minus index error, plus sun's semidiameter + moon's semidiameter + augmentation

49°55'58.1"	observed average
- 8'45"	sextant's index error
49°47'13"	
+ 15'47"	sun's semidiameter
+ 15'52"	moon's semidiameter
+ 00'10"	moon's augmentation
50°19'02"	apparent sun-moon separation

LONGITUDE from LUNAR DISTANCE from the SUN – SECOND SET of OBSERVATIONS
July 29, 1805 (page 4 of 5)

True separation of the Sun and the Moon, second set of observations – July 29, 1805

Patterson, Form V (modified)

A	=	Apparent separation of moon's center from sun's center	50°19'02"
B	=	Apparent altitude of moon's center	37°47'28"
C	=	Apparent altitude of sun's center	31°18'14"
D	=	½ (B + C) =	34°32'51"
E[1]	=	C ~ D =	03°14'37"
F	=	A ÷ 2 = 50°19'02" ÷ 2 =	25°09'31"
G	=	tangent-1 of (tangent D x cotangent E x tangent F) =	
	=	tangent-1 of (0.688502 x 17.645321 x 0.469682)	
	=	tangent-1 of 5.706096 =	80°03'35.2"
H[2]	=	F ± G =	54°54'04.2"
I[3]	=	F ± G =	105°13'06.2"
K	=	180 ÷ moon's horizontal parallax = 180 ÷ 58'12.7" = 180 ÷ 58.213' =	03.092093
L	=	180 ÷ cosecant B x tangent I x K =	
		180 ÷ (1.631895 x -3.675945 x 3.092093) = 180 ÷ 18.54871 = 9.70418 =	00°09'42"
M	=	refraction of I = [(983 x 25.86) ÷ 530] x cot (180°- I) [74°47'15"] =	00°00'14"
N	=	L - M = 9'42" - 14" = 1st correction =	00°09'28"
O[4]	=	A ± N = 50°19'02" - 9'28" =	50°09'34"
P	=	refraction and parallax of H for sun =	
	=	r = [(983 x 25.86) ÷ 530] x cot 54°54'04"; p = cosine H x 8.794	
	=	r = 33.7"; p = 5.1"; r - p = second correction = 28.6" =	00°00'29"
Q[5]	=	O ± P = 50°09'34" + 29"=	50°10'03"
R[6]	=	3rd correction, from Table XIII (Tables Requisite)	00°00'04"
S[7]	=	true distance = 50°10'02" + 4" =	50°10'07"
T[8]	=	preceding distance in Nautical Almanac (21h) =	48°45'25"
U[8]	=	following time in Nautical Almanac (24h) =	50°20'29"

Calculating the Longitude

48°45'25" x_1, 21h y_1,			
50°10'07"	>	23h40m22.3s	
50°20'29" x_2, 24h y_2		<u>16h16m03.7s</u>	average Local Apparent Time of observation
		07h24m18.6s	difference in time
		<u>x 15</u>	
		111°04'39" W	111°05" W (should be 111°30'40")

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1. The symbol ~ means “absolute value”.
2. Add (+) if C is greater than B, if not subtract (-); therefore F - G.
3. Subtract (-) if C is greater than B, if not add (+); therefore F + G.
4. Subtract (-) if H or I is greater than 90°; or H is greater than I; therefore A - N.
5. Add (+) if H or I is greater than 90°; or H is less than I; therefore O + P
6. In Table VII find the correction of moon's altitude, then in Table XIII under the nearest degree to Q at the top, find two numbers; one opposite to the nearest minute to moon's correction of altitude found as above, and the other opposite the nearest minute to first correction (N) and the difference of these two numbers will be the third correction. This correction may, without sensible error, be generally omitted.
7. Add (+) if Q is less than 90°, if not, subtract (-) = Q + R
8. T and U; These are to be found in Nautical Almanac from page 8th to page11th of the month opposite the day of the month and the sun or star from which the moon's distance was observed; taking out the two distances, which are next greater and next less than the true distance (S) calling that the <u>preceding distance</u> which comes first in the order of time and the other the <u>following distance</u>)

LONGITUDE CALCULATIONS

LONGITUDE FROM LUNAR DISTANCE from the SUN – SECOND SET of OBSERVATIONS
July 29, 1805 (page 5 of 5)

True separation of the Sun and the Moon, second set of observations – July 29, 1805

Jean Borda's Method (1787), modified to use RPN calculator

A	= ☽'s Apparent Altitude =	37°47'28"	m
B	= ☉'s Apparent Altitude =	31°18'14"	s
C	= Apparent Distance =	50°19'02"	d
D	= ☽'s True alt =	38°32'27.5"	M
E	= ☉'s True alt =	31°17'05"	S
F	= (A + B + C) ÷ 2 =	59°42'22"	(m + s + d) ÷ 2
G	= (A + B - C) ÷ 2 =	09°23'20"	(m + s - d) ÷ 2
H	= (D + E) ÷ 2 =	34°54'44.5"	(M + S) ÷ 2

I = $\cos^{-1}\sqrt{\ }$ secant A x secant B x cosine D x cosine E x cosine F x cosine G
= $\cos^{-1}\sqrt{\ }$ 1.265422 x 1.170380 x 0.782163 x 0.854597 x 0.504436 x 0.986604 =
= $\cos^{-1}\sqrt{\ }$ 0.492685 = $\cos^{-1}$ 0.701915 = 45.419131° = 45°25'09"

J = sine (I + H) = sine 80°19'55" = 0.985797
K = sine (I - H) = sine 10°30'23" = 0.182343
L = $\sin^{-1}\sqrt{\ }$ (J x K) = $\sin^{-1}\sqrt{\ }$ 0.179754 = $\sin^{-1}$ 0.423974 = 25.085713° = 25°05'09"

True Distance = 2L = 2 x 25.085713° = 50.171425° = 50°10'17"

48°45'25" x_1, 21h y_1;
50°10'17" >
50°20'29" x_2, 24h y_2

	23h40m41.2s	Greenwich Apparent Time
	16h16m03.7s	average Local Apparent Time of observation
	07h24m37.5s	difference in time
	x 15	degrees of longitude per hour
	111°09'23"	111°09' west longitude

Note: using a LAT of 16h16m08.7s (see Local Time - Summary) yields a longitude of 111°08'08" W

LONGITUDE CALCULATIONS

LONGITUDE from LUNAR DISTANCE from ANTARES
JULY 29, 1805 (page 1 of 5)

Observed time and Distance of ☽'s Western limb from α Antares, with Sextant ★ East

Time PM	Distance	Time PM	Distance
h m s	° ' "		
8 42 16	68 56 00	9 01 12	68 46 00
8 50 55	68 52 30	9 03 01	68 45 30
8 54 44	68 49 45	9 04 47	68 45 00
8 55 56	68 49 00	9 06 27	68 44 00
8 58 53	68 47 15	9 08 31	68 13 45

Calculations for Longitude from Lunar Distance from Antares – 1805 July 29

Average time by chronometer: 8:58:58.2 PM; average separation by sextant: 68°47'52.5". A plot of times and distances, however, shows that the time for data set No.1 probably should be 8h45m16s, not 8h42m16s and the distance for data set No. 10 should be 68°43'45", not 68°13'45". Owing to uncertainties regarding the correct values, however, these two data sets should not be used. In addition, the plot of times and distances for data sets 2 through 9 shows that from fata set No. 7 on, refraction is distorting the distances as the moon nears the horizon. The most reliable data sets are No. 2 through No. 6. The average of these values gives a chronometer time of 8h57m26.8s p.m. and a sextant angular distance of 68°48'20".

True Time of the Antares-Moon observation (see LOCAL TIME CALCULATIONS – Summary)

Ave chrono	fast LAT	True LAT	111°W	GAT	Eq of Time	GMT
20:57:26.8	00:33:00.9	20:24:25.9	+7h24m	27:48:25.9	+06m02.2s	03:54:28.1 30th

Sun - Moon Data from Nautical Almanac Calculated for time of Antares-Moon observation

RA Sun 29th	8h32m49.6s	Dec Sun 29th	+18°50'01"	SD Sun 25th	15'46.7"
RA Sun 30th	8h36m44.5s	Dec Sun 30th	+18°35'44"	SD Sun 32nd	15'47.5"
RA Sun obs	8h35m24.3s	Dec Sun obs	+18°40'37"	SD Sun obs	15'47.2"

RA Moon 29th 24h	174°48'	= 11h39m12s	Dec Moon 29th 24h	-3°08'
RA Moon 30th 12h	180°56'	= 12h03m44s	Dec Moon 30th 12h	-6°00'
RA Moon at obs	176°44'45"	= 11h46m59s	Dec Moon at obs	-4°02'32"

SD Moon 29th 24h	15'52"	HP Moon 29th 24h	58'12"	EqT 29th +6:03.3
SD Moon 30th 12h	15'44"	HP Moon 30th 12h	57'44"	EqT 30th +6:01.6
SD Moon at obs	15'49.46"	HP Moon at obs	58'03.1"	EqT obs +6:02.2

RA = right ascension HP = horizontal parallax
Dec = declination Eq = equation
SD = semidiameter obs = observation

Right Ascension (RA) and Declination (DEC) of Antares 1805 July 30

Tables Requisite: 1805 January 1, RA = 16h17m28s, annual variation = +3.64s
July 30 = 0.575 year x +3.64s per year = +2.1s + 16h17m28s = 16h17m30s
Tables Requisite: 1805 January 1: DEC = -25°59'05", annual variation = + 8.7"
July 30 = 0.575 year x +8.7" per year = +5.0" + -25°59'05" = -25°59'10"

LONGITUDE from LUNAR DISTANCE from ANTARES
JULY 29, 1805 (page 2 of 5)

Hour Angle of Antares and Moon

Antares		Moon
08h35m24.3s	Sun's Right Ascension at time of the observation	08h35m24.3s
08h24m25.9s	Local Apparent Time of the observation, pm =	08h24m25.9s
16h59m50.2s	Sum = Right Ascension of Meridian	16h59m50.2s
16h17m30s	Right Ascension at time of the observation	11h46m59s
00h42m20.2s	Hour Angle in hours	05h12m51.2s

True Altitude of Antares for the Antares-Moon observation

A	=	Latitude: meridian observations July 28 and 29 (to nearest 30" =	45°51' =	45.850000
B	=	Antares's Declination at the time of the observation =	-25°59'10" =	-25.986111
C	=	Antares' Hour Angle as an angle = 42m20.2s x 15 =	10°35'03" =	10.584167
D	=	cotangent^{-1} of (cotangent B x cosine C) =		
	=	cotangent^{-1} of (-2.051566 x 0.982986) = -2.016661 =	-26°22'31" =	-26.375397
E[1]	=	A ± D = 45°51' - -26°22'31" =	72°13'31" =	72.225397
F	=	cosecant^{-1} of (cosecant B x sine D x secant E)		
	=	cosecant^{-1} of (-2.282306 x -0.444251 x 3.275753) = -3.321339 = True Alt (Hc) =		17.522688° 17°31'22"

1. Add (+) if declination and latitude are of different signs or C is greater than 90°.

Apparent ("observed") Altitude of Antares

G_1	=	Refraction = {[(983 x 25.79 inches Hg) ÷ (460 + 60°)] ÷ 3600} x cot Hc (1st trial), then Ha afterwards	
		R° = [(26797 ÷ 520) ÷ 3600] x cotangent Hc, then Ha; G_1 = 0.013542 x cot Hc, Ha	
H	=	1) 17.522688° + (cotangent 17.522688° x 0.013542) = Ha1 =	17.565579°
		2) 17.565579° - (cotangent 17.565579° x 0.013542) =	17.522800°
		3) 17.522800° - 17.522688°(Hc) = 0.000112°	
		4) 17.565579° - 0.000112° = 17.565467° = Ha2	
		5) 17.565467° - (cot 17.565467° x 0.013542) =	17.522688° = Hc
		Therefore apparent altitude of Antares (Ha) =	17.565467°
		Δh★ = 17.565467° - 17.522688° = +0.042779°	17°33'56"

True Altitude of the Moon's Center for the Antares-Moon observation

A	=	Latitude: meridian observations July 28 and 29 (to nearest 30") =	45°51' =	45.850000
B	=	Moon's Declination at the time of the observation =	-4°02.5'	-4.041667
C	=	Moon's Hour Angle as an angle = 5h12m51.2s x 15 =	78°12'48"	78.213333
D	=	cotangent^{-1} (cotangent B x cosine C) =		
	=	cotangent^{-1} (-14.152754 x 0.204268) = -2.890958 =	-19°04'51"	-19.080813°
E[1]	=	A ± D = 45°51' - -19°04'51" =	64°55'51"	64.930813°
F	=	cosecant^{-1} of (cosecant B x sine D x secant E)		
	=	cosecant^{-1} of (-14.188039 x 0.326901 x 2.360092) = -10.946318 = True Alt (Hc)		5.241559° 5°14'30"

1. Add (+) if declination and latitude are of different signs or C is greater than 90°.

LONGITUDE CALCULATIONS

LONGITUDE from LUNAR DISTANCE from ANTARES
JULY 29, 1805 (page 3 of 5)

Apparent ("observed") Altitude of the Moon's Center at observation

G_1	=	Refraction = 0.013542 x cotangent Hc, Ha (see Antares, above)	
G_2	=	Parallax = moon's Horizontal Parallax (0.967532°) x cosine Hc, Ha	
H	=	1) 5.241559° + (cot 5.241559° x 0.013542) - (cos 5.241559° x 0.967532) =	4.425688° = Ha1
		2) 4.425688° - (cot 4.425688° x 0.013542) + (cos 4.425688° x 0.967532) =	5.215367°
		3) 5.215367° - 5.241559°(Hc) = -0.026192°	
		4) 4.425688° + 0.026192° = 4.451880° = Ha2	
		5) 4.451880° - (cot 4.451880° x 0.013542) + (cos 4.451880° x 0.967532) =	5.242558°
		6) 5.242558° - 5.241559°(Hc) = 0.000999°	
		7) 4.451880° - 0.000999° = 4.450881° = Ha3	
		8) 4.450881° - (cot 4.450881° x 0.013542) + (cos 4.450881° x 0.967532) =	5.241521° = Hc3
		9) 5.241521° - 5.241559°(Hc) = -0.000038°	
		10) 4.450881° + 0.000038° = 4.450919°	
		11) 4.450919° - (cot 4.450919° x 0.013542) + (cos 4.450919° x 0.967532) =	5.241559° = Hc
		Therefore moon's apparent altitude (Ha) =	4.450919°
		Δh☽ = 4.450919° - 5.241559° = - 0.790640°	4°27'03"

Moon's Augmentation from its Apparent Altitude in the Antares-Moon observation

Augmentation = (sine Ha x ☽'s SD) ÷ 60 = (sine 4°26'54" x 0.15'49.5") ÷ 60 = 0.000341° = 1.2"

Apparent Antares-Moon Distance

Observed distance, minus index error, minus (moon's semidiameter + augmentation) (far limb)

68°48'20"	sextant observed distance
- 8'45"	sextant's index error
68°39'35"	
- 0°15'49.5"	moon's semidiameter (far limb)
- 0°00'01"	augmentation (far limb)
68°23'44.5"	apparent separation of moon and Antares

LONGITUDE CALCULATIONS

LONGITUDE from LUNAR DISTANCE from ANTARES
July 29, 1805 (page 4 of 5)

True separation of Antares and the Moon – July 29, 1805

Patterson, Form V (modified)

A	=	Apparent separation of moon's center from Antares	68°23'44.5"
B	=	Apparent altitude of moon's center	04°27'03"
C	=	Apparent altitude of Antares	17°33'56"
D	=	½ (B + C) =	11°00'29.5"
E[1]	=	C ~ D =	06°34'53.8"
F	=	A ÷ 2 =	34°11'52.2"
G	=	tangent-1 of (tangent D x cotangent E x tangent F) =	
		tangent-1 of (0.194529 x 8.692432 x 0.677439) =	
		tangent-1 of 1.145500 =	48°52'47.7"
H[2]	=	F ± G =	83°04'39"
I[3]	=	F ± G =	14°40'54.5"
K	=	180 ÷ moon's horizontal parallax =180 ÷ 58'03.1" = 180 ÷ 58.05) =	03.100686
L	=	180 ÷ (cosecant B x tangent I x K) =	
	=	180 ÷ (12.886001 x 0.262005 x 3.100686) = 10.468546	
	=	180 ÷ 10.468546 = 17.194365' =	00°17'12"
M	=	refraction of I = [(983 x 25.86) ÷ 510] x cotangent 14°40'54.5" = 190" =	00°03'10"
N	=	L - M = 17'12" - 3'10" = 1st correction =	00°14'02"
O[4]	=	A ± N = 68°23'44.5" - 14'02" =	68°09'42.5"
P	=	refraction of H for star =	
	=	r = [(983 x 25.86) ÷ 510] x cotangent 83.04'39" =	00°00'06"
Q[5]	=	O ± P = 68°09'42.5" - 6"=	68°09'36.5"
R[6]	=	correction from Table XIII (Tables Requisite)	00°00'02"
S[7]	=	true distance = Q ± R = 68°09'36" + 2"	68°09'38"
T[8]	=	preceding distance in Nautical Almanac (27h) =	68°36'37"
U[8]	=	following time in Nautical Almanac (30h) =	66°55'14"

Calculating the Longitude

68°36'37" x_1; 27h y_1		
68°09'38" >	27h47m54.4s	
66°55'14" x_2; 30h y_2	20h24m25.9s	average Local Apparent Time of observation
	07h23m28.5s	difference in time
	x 15	
	110°52'08 " W	110°52 W (should be 111°30'40")

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1. The symbol ~ means "absolute value".
2. Add (+) if C is greater than B, if not subtract (-); therefore F + G.
3. Subtract (-) if C is greater than B, if not add (+); therefore F - G.
4. Subtract (-) if H or I is greater than 90°; or H is greater than I; therefore A - N.
5. Add (+) if H or I is greater than 90°; or H is less than I; therefore O - P
6. In Table VII find the correction of moon's altitude, then in Table XIII under the nearest degree to Q at the top, find two numbers; one opposite to the nearest minute to moon's correction of altitude found as above, and the other opposite the nearest minute to first correction (N) and the difference of these two numbers will be the third correction. This correction may, without sensible error, be generally omitted.
7. Add (+) if Q is less than 90°, if not, subtract (-) = Q + R
8. T and U; These are to be found in Nautical Almanac from page 8th to page11th of the month opposite the day of the month and the sun or star from which the moon's distance was observed; taking out the two distances, which are next greater and next less than the true distance (S) calling that the preceding distance which comes first in the order of time and the other the following distance)

LONGITUDE CALCULATIONS

LONGITUDE from LUNAR DISTANCE from ANTARES
JULY 29, 1805 (page 5 of 5)

True separation of Antares and the Moon – July 29, 1805

Jean Borda's Method (1787), modified to use RPN calculator

A	= ☽'s Apparent Altitude =	04°27'03"	m
B	= ★'s Apparent Altitude =	17°33'56"	s
C	= Apparent Distance =	68°23'44.5"	d
D	= ☽'s True alt =	05°14'30"	M
E	= ★'s True alt =	17°31'22"	S
F	= (A + B + C) ÷ 2 =	45°12'21.8"	(m + s + d) ÷ 2
G	= (A + B - C) ÷ 2 =	23°11'22.8"	(m + s - d) ÷ 2
H	= (D + E) ÷ 2 =	11°22'56"	(M + S) ÷ 2

I = $\text{cosine}^{-1}\sqrt{}$ secant A x secant B x cosine D x cosine E x cosine F x cosine G
= $\text{cosine}^{-1}\sqrt{}$ 1.003025 x 1.048908 x 0.995818 x 0953597 x 0.704559 x 0.919206 =
= $\text{cosine}^{-1}\sqrt{}$ 0.647031 = cosine^{-1} 0.804382 = 36.449364° = 36°26'57.7"

J = sine (I + H) = sine 47°49'54" = 0.741175
K = sine (I - H) = sine 25°04'02" = 0.423680
L = $\text{sine}^{-1}\sqrt{}$ (J x K) = $\text{sine}^{-1}\sqrt{}$ 0.314021 = sine^{-1} 0.560376 = 34.081785° =34°04'54"

True Distance = 2L = 2 x 34.081785° = 68.163571° = 68°09'49"

68°36'37" x_1, 27h y_1;

68°09'49" >	27h47m34.9s	Greenwich Apparent Time
66°55'14" x_2, 30h y_2	20h24m25.9s	average Local Apparent Time of observation
	07h23m09.1s	difference in time
	x 15	degrees of longitude per hour
	110°47'16"	110°47' west longitude

Note: using a LAT of 20h24m35.5s (see Local Time - Summary) yields a longitude of 110°44'52"W

MAGNETIC DECLINATION CALCULATIONS

MAGNETIC DECLINATION from COMPASS BEARING of the SUN – AM OBSERVATION
JULY 29, 1805 (page 1 of 2)

☉'s magnetic azimuth

Time by chrono-meter		Azimuth by circumferentor	Altitude of ☉'s lower limb with sextant
	h m s	°	° ' "
A.M.	8 48 09	N 85 E	73 00 00
	8 53 57	N 86 E	74 58 15

Calculations for Magnetic Declination from Compass Bearing of the Sun, AM Observation–July 29, 1805

Latitude

Average from sun's meridian altitude, 1805 July 28 and 29 (rounded to nearest 30") = 45°51'00"

Average Chronometer Time of Observations

8:48:09 + 8:53:57 = 17:42:06: ÷ 2 = 08:51:03

True Local Apparent Time of the Observation Average

Chronometer time of Local Noon, 1805 July 28 = 12:33:39.4
Chronometer time of Local Noon, 1805 July 29 as if the 28th (12h33m11.0s + 24h) 36:33:11.0
12h33m39.4s x_1, 12h y_1 and 36h33m11.1s x_2, 36h y_2
Solve for 32:51:03 (8:51:03 chronometer time of observation + 24h) = 32:17:47.6
32:17:47.6 - 24h = True Local Apparent Time of observation average = 08:17:47.6

Greenwich Apparent Time of Observation Average at Estimated 111° W Longitude

Time difference from 0° longitude to 111° W = 07:24:00
Local apparent time of observation average = 08:17:47.6
Estimated Greenwich Apparent Time of observation = 15:41:47.6

Sun's declination at Average Time of Observations

Sun's declination Greenwich Apparent Noon, 1805 July 29 = +18°50'01" N
Sun's declination Greenwich Apparent Noon, 1805 July 30 = +18°35'44" N
Sun's declination at 15:41:47.6 Greenwich Apparent Time = +18°47'49" N

Altitude of Sun's Center

73°59'07.5"	observed double altitude of sun's lower limb; average
- 0°08'45"	sextant's index error
73°50'22.5"	double altitude corrected for index error
÷ 2	divide by 2 (artificial horizon has doubled the angle read)
36°55'11	apparent altitude of sun's lower limb = H
- 0°01'04.9"	refraction (see Latitude, footnote 1)
+0°00'06.9"	parallax (see Latitude, footnote 2)
+0°15'47.2"	semidiameter
37°10'00"	true altitude of sun's center per this observation

Sun's Zenith Distance

Zenith distance = 90°- true altitude of center = 90°- 37°10'00" = 52°50'00"

Average Magnetic Bearing/Azimuth of the Sun

N85°E + N86°E = N85.5°E = Azimuth = 085.5°

MAGNETIC DECLINATION CALCULATIONS

MAGNETIC DECLINATION from COMPASS BEARING of the SUN – AM OBSERVATION
JULY 29, 1805 (page 2 of 2)

Sun's Azimuth at the Average Time of the AM Observation

$$\text{sine } \tfrac{1}{2} \text{ azimuth} = \sqrt{\frac{\text{cosine } \tfrac{1}{2} \text{ (Latitude + Zenith Distance + Declination) x sine } \tfrac{1}{2} \text{ (Lat + ZD - Dec)}}{\text{cosine Latitude x sine Zenith Distance}}}$$

Latitude	=	45°51'00"	
Zenith Distance	=	52°50'00"	
Sum	=	98°41'00"	98°41'00"
Declination	=	+18°47'49"	- 18°47'49"
		117°28'49"	79°53'11"
Divided by 2		÷ 2	÷ 2
		58°44'24.5"	39°56'35.5"

sine ½ azimuth = √ (cosine 58°44'24.5" x sine 39°56'35.5") ÷ (cosine 45°51'00" x sine 52°50'00")

= √ (0.518920 x 0.642028) ÷ (0.696539 x 0.796882)

= √ 0.333161 ÷ 0.555059

= √ 0.600227 = 0.774743

sine^{-1} 0.774743 = 50.781731° = ½ azimuth

azimuth = 50.781731° x 2 = 101°33'48"
Average Magnetic Azimuth = 085°30'
Average Magnetic Declination = 016°04' = 16° East rounded to nearest ½°

MAGNETIC DECLINATION CALCULATIONS

MAGNETIC DECLINATION from COMPASS BEARING of the SUN – PM OBSERVATION
JULY 29, 1805 (page 1 of 2)

Observed magnetic azimuth of the sun

	Time by chronometer	Azimuth by circumferentor	Altitude of ☉'s lower limb by sextant.
	h m s	°	° ' "
P.M.	5 07 47	S 72 W	55 44 30
	5 13 04	S 73 W	53 52 45

Calculations for Magnetic Declination from Compass Bearing of the Sun, AM Observation–July 29, 1805

Average bearing S72½°W = azimuth of 252.5°
Average from sun's meridian altitude, 1805 July 28 and 29 (rounded to nearest 30") = 45°51'00"
Ave Chronometer time of Observations (p.m.) = 5:07:47 + 5:13:04 = 10:20:51 ÷ 2 = 5:10:25.5 pm

True Local Apparent Time of Observation Average

Chronometer time of Local Noon 1805 July 28 = 12h33m39.4s
Chronometer time of Local Noon 1805 July 29 = 24h + 12h33m11.0s
12h33m39.4s x_1, 12h y_1 and 36h33m11.1s x_2, 36h y_2
Solve for 41:10:25.5 (17:10:25.5 chronometer time of observation average + 24h) = 40:37:20.0
40:37:20.0 - 24 h = True Local apparent Time of observation average = 16:37:20.0

Greenwich Apparent Time of Observation Average at Estimated 111° W Longitude

Time difference from 0° longitude to 111° W = 07:24:00
Local Apparent Time of observation average = 16:37:20
Greenwich Apparent Time of observation average = 24:01:20

Sun's Declination at Average Time of Observation

Sun's declination Greenwich Apparent Noon, 1805 July 29: 18°50'01" N
Sun's declination Greenwich Apparent Noon, 1805 July 30: 18°35'44" N
Sun's declination at 24:01:20 Greenwich Apparent Time = 18°42'52" N

Altitude of the Sun's Center

54°48'37.5"	double altitude of sun's lower limb; average
- 0°08'45"	sextant's index error
54°39'52.5"	double altitude
÷ 2	
27°10'56"	apparent altitude = H
- 0°01'30"	refraction (see Latitude, footnote 1)
+0°00'08"	parallax (see Latitude, footnote 2)
+0°15'47"	semidiameter
27°34'21"	true altitude of sun's center per this observation

Sun's Zenith Distance

Zenith distance = 90°- true altitude of sun's center = 90°- 27°34'21" = 62°25'39"

MAGNETIC DECLINATION CALCULATIONS

MAGNETIC DECLINATION from COMPASS BEARING of the SUN – PM OBSERVATION
JULY 29, 1805 (page 2 of 2)

Sun's Azimuth at the Time of the Observation Average

$$\text{sine } \tfrac{1}{2} \text{ azimuth} = \sqrt{\frac{\text{cosine } \tfrac{1}{2} \text{ (Latitude + Zenith Distance + Declination) x sine } \tfrac{1}{2} \text{ (Lat + ZD - Dec)}}{\text{cosine Latitude x sine Zenith Distance}}}$$

Latitude	=	45°51'00"	
Zenith Distance	=	62°25'39"	
Sum	=	108°16'39"	108°16'39"
Declination	=	+18°42'52"	- 18°42'52"
		126°59'31"	89°33'47"
Divided by 2		÷ 2	÷ 2
		63°29'45.5"	44°46'53.5"

sine ½ azimuth = √ (cosine 63°29'45.5" x sine 44°46'53.5") ÷ (cosine 45°51'00" x sine 62°25'39")

= √ (0.446261 x 0.704405) ÷ (0.696539 x 0.886426)

= √ (0.314348 ÷ 0.617430)

= √ 0.509124 = 0.713529

sine^{-1} 0.713529 = 45.522782° = ½ azimuth

azimuth = 45.522782 x 2 = 91.045564° (from north because sun is west in p.m.)

true azimuth	= 360° - 91.045564 =	268°57'16"	
Average Magnetic Azimuth =		252°30'	
Average magnetic Declination =		016°27'16" =	16½ ° East

Average of Sun observations: a.m. 16.06°E + p.m. 16.45°E = 16¼° East

MAGNETIC DECLINATION CALCULATIONS

MAGNETIC DECLINATION from COMPASS BEARING of POLARIS
JULY 29, 1805 (page 1 of 2)

Observed the azimuth of the Pole Star

	Time by chronometer	Azimuth by circumferentor
	h m	
P.M.	9 27	N13°W

Calculations for Magnetic Declination from Compass Bearing of Polaris – July 29, 1805

Magnetic bearing of Polaris at the Time of Observation = N 13° W

True Local Apparent Time of the Observation

Chronometer time of Local Noon, 1805 July 28 12h33m39.4s
Chronometer time of Local Noon, 1805 July 29 24h + 12h33m11.0s
Chronometer loss per day on Local Apparent Time 28.4s = 1.18 seconds per hour

Chronometer time of observation (21:27:00) - chronometer time of noon (12:33:11) = 8.9h (later)
8.9h x loss of 1.18s per hour = 10.4s extra loss
Chronometer fast at noon July 29th (33m11.0s) - loss since noon (10.4s) = 33m00.4s fast at observation
True Local Apparent Time of observation = chronometer time (21:27) - fast (33m00.4s) = 20:53:59.6

12:33:39.4 x_1,12 y_1; 36:33:11.0 x_2; 36 y_2; solve for 45:27:00 (21:27 + 24h) = 44:53:59.6
44:53:59.5 - 24h = True Local Apparent time of observation = 20:53:59.6

Estimated Greenwich Apparent Time of the Observation

Estimated longitude = 111° W
Time difference from 0° longitude to 111° W = 111° ÷ 15 = 07:24:00.0
Local Apparent Time of observation = 20:53:59.6
Estimated Greenwich Apparent Time of observation = 28:17:59.6
Hours since Greenwich noon 16:17:59.6

Right Ascension of the Sun at the Time of Observation

Sun's Right Ascension Greenwich Apparent Noon, 1805 July 30 = 8h36m44.5s
Sun's Right Ascension Greenwich Apparent Noon, 1805 July 29 = 8h32m49.6s
difference per day 0h03m54.9s
÷ 24h
change per hour 9.79 seconds
RA at obs = 9.79 seconds x 16h18m since noon = 2m39.6s + 8h32m49.6s = 8h35m29.2s
Look-up table 9.8 sec at 16h = 156.8s + 18m = 3s = 159.8s = 2m39.8s + 8h32m49.6s = 8h35m29.4s
Sun's RA at obs also = 12x_1, 8:32:49.6y_1; 36x_2 , 8:36:44.5y2; solve for 28:17:59.6 = 8h35m29.1s

Right Ascension (RA) and Declination (DEC) of Polaris for the Date of Observation

From Tables Requisite 1805 Jan 1; RA = 0h53m25s, annual variation = +12.89s
July 30 = 0.575 year x +12.89s variation per year = +7.4s = 00h53m32s

From Tables Requisite 1805 Jan 1: DEC = 88°15'50", annual variation = +19.6"
July 30 = 0.575 year x +19.6" variation per year = +11.3" 88°16'01"

MAGNETIC DECLINATION CALCULATIONS

MAGNETIC DECLINATION from COMPASS BEARING of POLARIS
JULY 29, 1805 (page 2 of 2)

Hour Angle of Polaris

Right Ascension Sun at Observation =	08h35m29.1s	
Local Apparent Time of observation PM =	08h53m59.6s	
Right Ascension of the Meridian =	17h29m28.7s	17h29m29s
Right Ascension of Polaris =		00h53m32s
Hour Angle of Polaris in time =		16h35m57s
Hour Angle of Polaris in degrees =		248°59'15s

True Altitude of Polaris at Observation by Patterson's Problem 4th, Example 3; Form IV A

A	=	Latitude: mean of observations of 28 and 29 July 1805 (to nearest 30") =	45°51'00"
B	=	Declination of Polaris at observation =	88°16'01"
C	=	Hour Angle of Polaris, in degrees, at observation =	248°59'15"
D	=	cotangent^{-1} (cotangent B x cosine C) =	
	=	cotangent^{-1} (0.030257 x -0.358572) = -0.010849 =	-89.378410°
E	=	A ± D = 45.850000° - -89.378410° =	135.228410°
F	=	cosecant^{-1} (cosecant B x sine D x secant E) =	
	=	cosecant^{-1} (1.000458 x 0.999941 x -1.408609) = 1.409171 = True Alt (Hc) =	45.205394°
Zenith Distance of Polaris = 90° - True Altitude =			44°47'41"

Azimuth of Polaris at the Time of Observation

$$\text{sine ½ azimuth} = \sqrt{\frac{\text{cosine ½ (Latitude + Zenith Distance + Declination) x sine ½ (Lat + ZD - Dec)}}{\text{cosine Latitude x sine Zenith Distance}}}$$

Latitude	=	45°51'00"	
Zenith Distance	=	44°47'41"	
Sum		90°38'41"	90°38'41"
Declination	=	+88°16'01"	-88°16'01"
		178°54'42"	2°22'40"
		÷ 2	÷ 2
		89°27'21"	1°11'20"

sine ½ azimuth = √ (cosine 89°27'21" x sine 1°11'20") ÷ (cosine 45°51'00" x sine 44°47'41")

= √ (0.009497 x 0.020749) ÷ (0.696539 x 0.704569)

= √ 0.000197 ÷ 0.490760

= √ 0.000402 = 0.020038

sine^{-1} 0.020038 = 1.148186° = ½ azimuth

azimuth = 1.148186° x 2 = 2.296372° = 2°17'47"

Observed Magnetic Bearing, 1805 July 29	=	N13°W
Calculated Bearing (rounded)	=	N 2¼°E
Magnetic Declination per this observation	=	15¼° East
Magnetic Declination averaged from AM and PM with sun Observations	=	16¼°East

Declination if Greenwich Apparent Time 28:18:10.1 (LAT 20:54:10.1) = N13°W + 2°17'04"E = 15¼°E